ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : JAN / FEB 2010

REGULATIONS : 2008

FIRST SEMESTER

080030001 - MATHEMATICS I

(COMMON TO ALL BRANCHES)

TIME : 3 Hours **Max.Marks : 100**

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Two of the eigen values of $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the eigen values of A^{-1}.

2. Find the sum of the squares of the eigen values of $\begin{bmatrix} 1 & 7 & 5 \\ 0 & 2 & 9 \\ 0 & 0 & 5 \end{bmatrix}$

3. State Cayley – Hamilton Theorem.

4. State the nature of quadratic form $2xy+2yz+2zx.$

(or)

4. Write the matrix of the quadratic form $Q(x,y)=3x^2+2y^2-4xy$

5. Find the equation of the sphere whose diameter is the join of (2,-3,1) and (1,-2,-1)

6. Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$

7. Find the equation to the cone whose vertex is at the origin and the base the circle $x=a, y^2+z^2=b^2$.

(or)

7. Find the centre and radius of a great circle on the sphere $x^2+y^2+z^2-2x-4y+6z-2=0$

8. Define a right circular cylinder.

9. Find the radius of curvature for $y=e^x$ at (0,1)

10. Find the centre of curvature of y= x^2 at the origin.

11. Find the radius of curvature of the curve $x=a\cos\theta, y=b\sin\theta$ at any point 'θ'.

12. Find the envelope of the family of straight lines $y=mx+\frac{a}{m}$.

13. If $u=\sin^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\tan u.$

(or)

13. If $u=\frac{y}{z}+\frac{z}{x}+\frac{x}{y}$, give the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$

14. Find $\frac{dy}{dx}$, *if* $x^3+y^3=3ax^2y$.

15. Find the stationary points of $f(x,y)=x^3+y^3-3x-12y+20.$

(or)

15. Give the Maclaurins series expansion of $f(x,y)=xy^2$

16. Find the Jacobian $\frac{\partial(r,\theta)}{\partial(x,y)}$ if $x=r\cos\theta, y=r\sin\theta.$

17. Evaluate $\int_0^1 \int_0^x dy\ dx$

18. Change the order of Integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\ dx$

(or)

18. Change the order of integration in $\int_0^1 \int_0^y f(x,y) dx dy$

19. Evaluate $\int_0^{\pi} \int_0^{\sin\theta} r\, dr\ d\theta$

20. Evaluate $\int_0^2 \int_1^3 \int_1^2 x y^2 z\ dz\, dy\, dx$

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a Find the eigen values and eigen vectors of $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ 6

b Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and hence find A^{-1} 6

22. a Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0,\ x+y+z=3$ as a great circle 8

b Find the equation to the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which has a semi vertical angle of 30°. 4

23. a Find the radius of curvature at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3+y^3=3axy$ 6

b Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 6

24. a Expand $e^x \cos y$ in powers of x and y as far as the terms of the 3rd degree 6

b A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction 6

25. a Transform into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dy dx$ 6

b Find the volume of the tetrahedron bounded by the plane x + y + z = 1 and the coordinate planes. 6

26. Diagonalise the matrix using orthogonal transformation $A = \begin{pmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{pmatrix}$

27. a Find the equation of the circle of curvature at (c,c) on $xy = c^2$. 6

b If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ 6

28. a Find the equation to the right circular cylinder of radius 4 and whose axis is the line $\frac{x}{2} = \frac{y}{1} = \frac{z}{-2}$ 6

b Change the order of integration and evaluate $\int_0^a \int_x^a (x^2 + y^2)\, dy\, dx$. 6

*****THE END*****